TRINITY COLLEGE

Mathematics Specialist Units 3,4
Test 4 2018

Section 1 Calculator Free
Integration and Applications of Integration

STUDENT'S NAME ____

DATE: Friday 20 July **TIME:** 36 minutes **MARKS**: 36

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

Determine $\int x\sqrt{x-3}dx$ let $u = x-3$

2. (9 marks)

(a) $\int e^{x}\sin(e^{x+3})dx$ let $u = e^{x+3}$ [4]

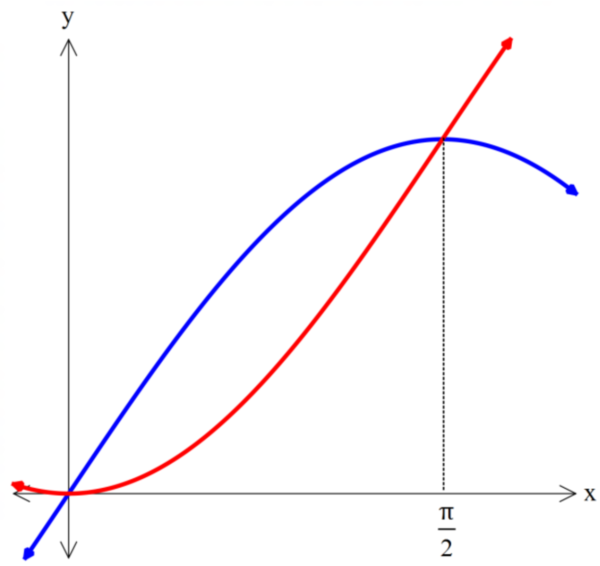
(b) $\int_{2}^{e} \frac{1}{x\ln\sqrt{x}}\,dx$ let $u = \ln\sqrt{x}$ [5]

3. (9 marks)

(a) $\int \sin^3 t \cos^2 t \, dt$ let $x = \cos t$ [5]

(b) $\int \frac{8x-7}{2x-3} dx$ [4]

4. (7 marks)



Shown above are the functions $y = \sin x$ and $y = 1 - \cos x$. The area enclosed between the two graphs is rotated about the x-axis. Determine the exact volume of the solid created.

5. (7 marks)

(a) Determine $\int \frac{4x+2}{x^2+x-2}\,dx$ [2]

(b) Determine $\int \frac{2x+10}{x^2+x-2}\,dx$ [5]

TRINITY COLLEGE

Mathematics Specialist Units 3,4
Test 2 2018

Section 2 Calculator Assumed

Integration and Applications of Integration

STUDENT'S NAME ____

DATE: Friday 20 July **TIME:** 14 minutes **MARKS**: 14

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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6. (6 marks)

The function $y = f(x)$ is a continuous curve in the first quadrant. Some values are shown in the table below.

x	0	1	2	3	4	5
$f(x)$	8.9	11.7	14.3	16.6	18.6	20.4

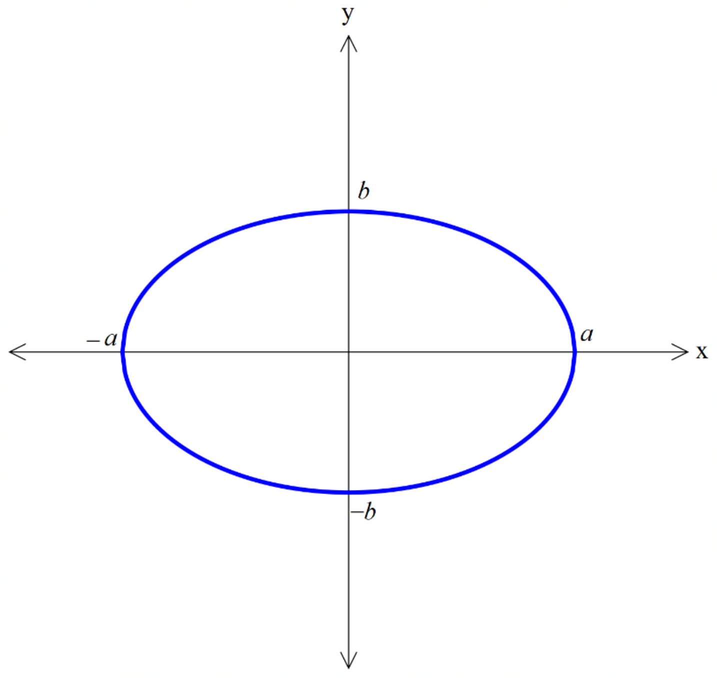
(a) Determine an approximation for the area between the curve and the x-axis for $1 \leq x \leq 4$ by summing the areas of trapeziums. [4]

(b) Is the estimation in (a) less than or greater than the exact area? Justify your answer. [2]

7. (8 marks)

The ellipse drawn below is centred at the origin and has equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ or

$y=\pm\frac{b}{a}\sqrt{a^2-x^2}$ for $a,b>0$



(a) Show the area of the ellipse is given by $\frac{4b}{a}\int_0^a \sqrt{a^2-x^2}\,dx$ [2]

(b) Using the substitution $x = a\sin\vartheta$, show the exact area of the ellipse is $ab\pi$. [6]